SparQ User Manual V0.6

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SFB/TR 8 Report No. 007-07/2006
Report Series of the Transregional Collaborative Research Center SFB/TR 8 Spatial Cognition
Universität Bremen / Universität Freiburg

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SFB/TR 8 Spatial Cognition - Project R3-[Q-Shape]

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July 28, 2006

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1 Introduction

SparQ is a toolbox for representing space and reasoning about space based on qualitative spatial relations. It is developed within the R3-[Q-Shape] project of the SFB/TR 8 Spatial Cognition funded by the Deutsche Forschungsgemeinschaft (DFG). SparQ is based on results from the qualitative spatial reasoning (QSR) community which consists of researchers from a various disciplines including computer science, artificial intelligence, geography, philosophy, psychology, and linguistics. During the last two decades, a multitude of formal calculi over sets of spatial relations (like 'overlaps', 'left-of', 'north-of') have been proposed, focusing on different aspects of space (mereotopology, orientation, distance, etc.) and dealing with different kinds of objects (points, line segments, extended objects, etc.). SparQ aims at making these qualitative spatial calculi and the developed reasoning techniques available in a single homogeneous framework that can easily be included into AI applications.

The current version of SparQ provides the following services (which will be thoroughly explained later in this manual):

qualification — A quantitative geometric description of a spatial configuration can be transformed into a qualitative description employing one of the supported spatial calculi.

computing with relations — The operations defined in a calculus (intersection, union, complement, converse, composition, etc.) can be employed to perform algebraic computations with the spatial relations from one of the supported calculi.

constraint reasoning — Techniques for solving relational constraint satisfaction problems (CSPs) based on the supported calculi can be used to check the consistency of spatial information, determine if a particular relation can hold between certain objects, or derive a concrete possible scenario from under-specified information.

In its current state, SparQ is a set of C libraries, calculi specifications, and a main program written in Lisp. It is available for POSIX systems (installation is explained in Section 2). The main program can either be used directly from the console or included into own applications via TCP/IP streams[1]. The details of using SparQ are explained in Section 5.

[1]Using the libraries directly is neither documented nor recommended as their interfaces have not been unified so far and not all parts of the toolbox come in the form of libraries.

Section 3 offers a small introduction to qualitative spatial calculi and qualitative spatial reasoning in general and introduces the relevant terms required for working with SparQ. The spatial calculi currently supported by SparQ are documented in Section 4 of this manual. SparQ is designed in a way that makes it easy to specify and integrate new calculi.

For an up-to-date list of supported calculi and the newest version of this manual, please visit our website (`http://www.sfbtr8.uni-bremen.de/project/r3/sparq/`). For questions or feedback, please send us an e-mail to the address below. We are always interested in suggestions for improvement and in hearing about your experience with SparQ.

The R3-[Q-Shape] team
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2 Installation of SparQ

SparQ is written for POSIX systems. Its functionality is continuously tested on Linux, Solaris and Mac OS X, but it should run on any Unix system without problems. At the moment, there is no Windows version available. If you want to run SparQ on Windows, please contact us.

2.1 Prerequisites

SparQ is currently not available in binary versions. To install and use SparQ, you need

- gcc and g++, version 2.95 or higher
- Steel Bank Common Lisp (SBCL)[1], version 0.9.10 or higher

2.2 Obtaining the source code

The source code of the actual version of SparQ and appropriate documentation is available at the SparQ-homepage (`http://www.sfbtr8.uni-bremen.de/project/r3/sparq/`).

2.3 Building the binaries

To build a running version of SparQ, unpack the source code package, enter the newly created SparQ directory (called `sparq-<version>`) and run

```
./configure
```

Usually, no errors should occur and you should be able to build the SparQ executables by running

```
make
```

All executables will be installed within the SparQ directory. If you encounter any problems during the build process, please contact the authors.

[1]`http://sbcl.sourceforge.net/`

3 Reasoning with Qualitative Spatial Relations

In this section, we provide a brief introduction on qualitative spatial reasoning and explain the most important terms required when dealing with qualitative spatial calculi in SparQ. For more in-depth introductions to the field, we refer to Cohn and Hazarika (2001), Cohn (1997), Ladkin and Reinefeld (1992), Ladkin and Maddux (1994), Düntsch (2005), and the references provided for particular calculi in Section 4.

3.1 What is a qualitative spatial calculus?

A qualitative calculus consists of a set of relations between objects from a certain domain and operations defined on these relations. Let us start with an easy example: the spatial version of the Point Algebra (PA) (Vilain et al., 1989). Imagine, we are being told about a boat race on a river by a friend on the phone[1]. We can model the river as an oriented line and the boats of the 5 participants A,B,C,D,E as points moving along the line (see Fig. 3.1). Thus, our domain (the set of spatial objects considered) is the set of all 1D points.

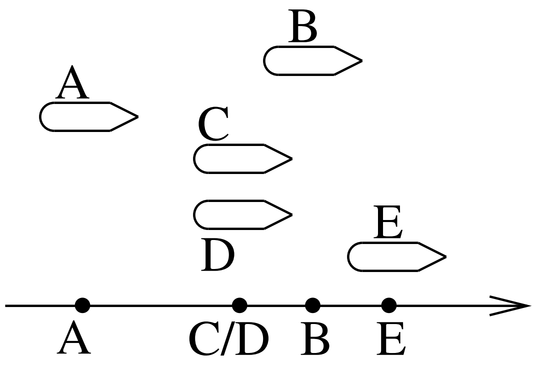


Figure 3.1: A possible situation in a boat race which can be modeled by 1D points on an oriented line and be described by qualitative relations from the Point Algebra.

We now distinguish three relations between objects from our domain. A boat can be

[1]This example has been borrowed from Ligozat (2005).

ahead of another boat, *behind* it, or on the *same* level. These relations can be used to formulate knowledge about the current situation in the race. For instance, our friend tells us the following:

1. A is *behind* B
2. E is *ahead* of B
3. A is *behind* C
4. D is on the *same* level as C
5. A is *ahead* of D

From this information we are able to conclude that our friend must have made an error, probably confusing the names of the participants: We know that A is *behind* C (sentence 3) and D is *behind* A (conversion of sentence 5). From composing these two facts it follows that C and D cannot be on the *same* level which contradicts sentence 4.

On the other hand, only taking the first three sentences into account, we can conclude that E is also *ahead* of A by composing the facts A is *behind* B (sentence 1) and B is *behind* E (conversion of sentence 2). However, this information is not sufficient to derive the exact relation between C and E, as C can either be *ahead*, *behind* or on the *same* level as E.

The calculus, in this case the PA, defines a set of base relations (*ahead*, *behind*, and *same*) and provides the elementary reasoning steps in the form of operations defined over the base relations. In our small example, the applied operations were conversion, which given the operation between x and y returns the relation between y and x (thus the converse of *ahead* is *behind*), and composition which takes the relations holding between X & Y and Y & Z and returns the relation holding between X & Z (e.g. composition of *ahead* and *ahead* is *ahead*).

Often the result of operations like the composition operation is not a single base relation but the union of more than one. For instance, knowing that X is *ahead* of Y and Y is *behind* Z yields the union of *ahead*, *behind*, and *same*. Because of this, the set of relations considered in a spatial calculus is not just the set of base relations, but the set of all unions of base relations including the empty set and the union of all base relations (the universal relation). All operations of the calculus are then defined for all unions of base relations: For example, we can apply conversion to the information that X is either *ahead* or at the *same* level as Y to infer that Y is either *behind* or at the *same* level as X.

3.2 Constraint networks, consistency, and consistent scenarios

A spatial configuration of a finite set of objects from the domain as given by sentences 1–5 can be described as a *constraint network* as shown in Fig. 3.2. It consists of a variable for each object represented by the nodes of the network and edges labeled with relations from the considered calculus denoted as sets of base relations. For instance, sentence 1 is represented by the edge going from A to B labeled with {*behind*}. If no edge connects two nodes, this corresponds to an edge labeled with the universal relation U, which is usually omitted.

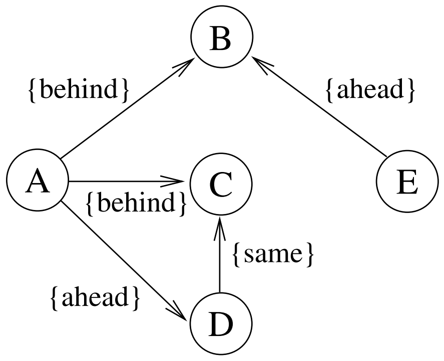


Figure 3.2: The situation described by sentences 1–5 as a constraint network.

As we have seen in our example, the information given in a constraint network can be inconsistent. This means, no objects from the domain can be assigned to the variables so that all the constraints given by the spatial relations annotated to the edges are satisfied. If, on the other hand, such an assignment can be found, the constraint network is said to be *consistent* or *satisfiable* or *realizable*. Determining whether a constraint network is consistent is a fundamental problem of qualitative spatial reasoning. Special techniques for determining consistency based on the operations of the calculus (especially the composition operation) have been developed. However, it is important to note that the soundness of these methods depends on the properties of the calculus at hand and are often still subject of ongoing investigations. For more details on this issue, we refer to Renz and Ligozat (2005) and the literature on individual calculi listed in Section 4.

A constraint network in which every constraint between two variables is a base relation is called *atomic* or a *scenario*. This means all spatial relations between two objects are completely determined with respect to the employed calculus and the remaining questions is if the network is consistent or not. However, if a constraint network contains relations that are not base relations like in Fig. 3.3(a), we might also be interested in finding a scenario that is a *refinement* of the original network (meaning it has been derived by removing individual base relations from the sets annotated to the edges) and that is consistent. Fig. 3.3(b) shows such a *consistent scenario* for the network in

Fig. 3.3(a). If such a consistent scenario can be found, we also know that the original network is consistent. Otherwise, we know it is inconsistent. Of course, it is possible that more than one consistent scenario exists for a given constraint network and we might be interested in finding only one or all of these. An alternative consistent scenario is depicted in Fig. 3.3(c).

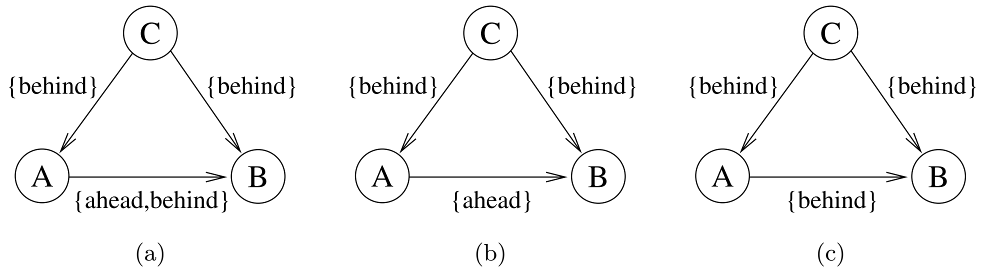


Figure 3.3: A non-atomic constraint network (a) with possible consistent scenarios (b) and (c).

The problems of determining consistency and finding consistent scenarios are subsumed under the term *constraint-based reasoning* throughout this text.

3.3 Binary and ternary spatial calculi

After giving a rather intuitive introduction to qualitative spatial calculi, we want to give a more formal definition of a spatial calculus and especially the operations that need to be defined for a calculus. This set of operations differs depending on whether we are dealing with a *binary calculus* (like the PA) in which all relations relate two objects or with a *ternary calculus* in which all relations relate three objects.

As we have seen an n-ary qualitative spatial calculus consists of:

1. a domain D which contains the considered spatial objects (D could for instance be the $\mathbb{R}^2$ when we are considering point objects in the plane)

2. a finite set $\mathcal{BR}$ of n-ary relations on D which are called base relations (typically the set of base relations is jointly exhaustive and pairwise disjoint which means that exactly one of the base relations holds for each n-tuple from D)

3. a finite overall set of relations $\mathcal{R}$ that is derived from $\mathcal{BR}$ by taking all possible unions of base relations (as we have seen, it is common to write these relations as sets of base relations, in which case $\mathcal{R} = 2^{\mathcal{BR}}$)

4. a set of operations closed over $\mathcal{R}$

Let us now turn to the operations that need to be defined. As all relations for a given calculus are subsets of tuples from the same Cartesian product, the three set operations *union*, *intersection*, and *complement* are already defined for every calculus independent of its arity:

Union: $R \cup S = \{ \ x \mid x \in R \vee x \in S \ \}$

Intersection: $R \cap S = \{ \ x \mid x \in R \wedge x \in S \ \}$

Complement: $\overline{R} = U \setminus R = \{ \ x \mid x \in U \wedge x \notin R \ \}$

where R and S are relations from $\mathcal{R}$.

Using our set notation for relations from $\mathcal{R}$, these set operations directly transfer to these sets. For example, the intersection of the PA relations $\{behind, same\}$ and $\{same, ahead\}$ is the relation $\{same\}$ and the complement of $\{same\}$ is $\{behind, ahead\}$. The other operations depend on the arity of the calculus.

Operations of binary calculi

For binary calculi, two more operations are utilized. These are the *converse* and *composition* operations, we have already encountered above:

Converse: $R^{\smile} = \{ \ (y,x) \mid (x,y) \in R \ \}$

Composition: $R \circ S = \{ \ (x,z) \mid \exists y \in D : ((x,y) \in R \wedge (y,z) \in S) \ \}$

Again, here are two examples from the PA: The converse of $\{behind\}$ is $\{ahead\}$ and the composition of $\{ahead\}$ and $\{ahead, same\}$ is $\{ahead\}$ (if A is ahead of B and B is ahead or at the same level as C than A has to be ahead of C).

For some calculi, the composition operation defined does not comply to the "strong" definition above, as this would result in a composition that is not closed for the set of relations $\mathcal{R}$ (and no finite set of relations including the base relations exists for which this is the case). In this case, the used composition operation is the "weak" composition that takes the union of all base relations that have a non-empty intersection with the result of the strong composition:

Weak composition: $R \circ_{weak} S = \{ \ d \mid T \in \mathcal{BR} \wedge d \in T \wedge T \cap (R \circ S) \neq \emptyset \ \}$

For some calculi, it is not yet known whether the defined composition operation is "strong" or not.

3.4 Ternary calculi and their operations

While there is only one possibility to permute the two objects of a binary relation which leads to the converse operation, there exist 5 such permutations for the three objects of a ternary relation. This results in the following 5 operations introduced in Zimmermann and Freksa (1996):

Inverse: $\mathrm{INV}(R) = \{\ (y, x, z) \mid (x, y, z) \in R\ \}$

Short cut: $\mathrm{SC}(R) = \{\ (x, z, y) \mid (x, y, z) \in R\ \}$

Inverse short cut: $\mathrm{SCI}(R) = \{\ (z, x, y) \mid (x, y, z) \in R\ \}$

Homing: $\mathrm{HM}(R) = \{\ (y, z, x) \mid (x, y, z) \in R\ \}$

Inverse homing: $\mathrm{HMI}(R) = \{\ (z, y, x) \mid (x, y, z) \in R\ \}$

It is in general not needed to specify all these operations as some can be expressed by others, but they are all available in SparQ nonetheless. Composition for ternary calculi is defined accordingly to the binary case:

Composition: $R \circ S = \{\ (w, x, z) \mid \exists y \in D : ((w, x, y) \in R \wedge (x, y, z) \in S)\ \}$

Other ways of composing two ternary relations can be expressed as a combination of the unary permutation operations and the composition (Scivos and Nebel, 2001) and thus do not have to be defined separately and are also not accessible individually in SparQ. The definition of weak composition is identical to the binary case.

As we have not introduced a ternary calculus so far, we resign from giving any examples here, but some can be found later in Section 5.

3.5 Path-consistency and backtracking search

Determining consistency of a constraint network in which the constraints are given as qualitative spatial relations from a particular calculus, is a particular instance of a *constraint satisfaction problem* (CSP). Unfortunately, the domains of our variables are typically infinite (e.g. the set of all points in the plane) and thus backtracking over all the values of the domain cannot be used to determine consistency.

The techniques developed for relational constraint problems are instead based on weaker forms of consistency called *local consistencies* which can be tested or enforced based on the operations of the calculus and which are under particular conditions sufficient to decide consistency.

One important form of local consistency is *path-consistency* which (in binary CSPs) means that for every triple of variables each consistent evaluation of the first two variables can be extended to the third variable in such a way that all constraints are satisfied. In the best case, path-consistency decides consistency for a given calculus. This means, that if we can make the network path-consistent by possibly removing some base relations from the constraints without ending up with the empty relation, we know that the original network is consistent. If this cannot be achieved, the network has to be inconsistent. Unfortunately, it is usually not the case that path-consistency decides consistency.

However, sometimes path-consistency is sufficient to decide consistency at least for a subset $\mathcal{S}$ of the relations from $\mathcal{R}$, for instance the set of base relations. On the one hand, this means that whenever our constraint networks only contains labels which are base relations, we again can use path-consistency as a criterion to decide consistency. On the other hand, if the subset $\mathcal{S}$ exhaustively splits $\mathcal{R}$ (which means that every relation from $\mathcal{R}$ can be expressed as a union of relations from $\mathcal{S}$), this at least allows to formulate a backtracking algorithm to determine consistency by recursively splitting the constraints and using path-consistency as a decision procedure for the resulting CSPs with constraints from $\mathcal{S}$ (Ladkin and Reinefeld, 1992).

To enforce path-consistency, syntactic procedures called *algebraic closure algorithms* have been developed that are based on the operations of the calculus (the composition operation in particular) and work in $O(n^3)$ time for binary calculi and $O(n^4)$ for ternary calculi where n is the number of variables. But again, we have to note that these syntactic procedures do not necessarily yield the correct results with respect to path-consistency as defined above. Whether algebraic closure coincides with path-consistency needs be investigated for each calculus individually and we again refer to the literature listed in the individual calculus descriptions in Section 4.

4 Supported Calculi

In this section, we briefly describe the spatial calculi currently supported by SparQ. Some calculi are actually calculi families for which a set of *calculus parameters* needs to be specified in order to obtain a particular calculus instance. For instance, for the $\mathcal{OPRA}_m$ calculus the granularity (number of partitioning lines) has to be specified as a calculus parameter.

Each calculi description in this section starts with a box summarizing the main characteristics of the considered calculus. The meaning of the entries in the box are explained below:

short name — the name used in SparQ to refer to this calculus

calculus parameters — the parameters that need to be specified whenever using this calculus

arity — the arity of the relations of this calculus (binary or ternary)

entity type — the spatial entities related in this calculus (2D points, oriented 2D points, line segments dipoles), etc.)

description — a short description of the calculus

base relations — a naming scheme or list of the base relations of the calculus

references — references to literature about this calculus

remarks — special remarks concerning the calculus

A quick overview on the implemented calculi is given in Tab. 4.1 which also classifies the calculi according to their arity (binary, ternary), their domain (points, oriented points, line segments, regions), and the aspect of space modeled (orientation, distance, mereotopology).

	arity		domain				aspect of space		
Calculus	binary	ternary	point	or. point	line seg.	region	orient.	dist.	mereot.
FFC/$\mathcal{LR}$		✓	✓				✓		
SCC		✓	✓				✓		
DCC		✓	✓				✓		
$\mathcal{DRA}_c$	✓				✓		✓		
$\mathcal{OPRA}_m$	✓			✓			✓		
RCC-5/8*	✓					✓			✓
Interval algebra*	✓								

Table 4.1: The calculi currently included in SparQ. For entries marked by a '$*$' no qualifier is available yet. Allen's Interval Algebra is originally intended for temporal reasoning, but can also be interpreted spatially.

4.1 FlipFlop calculus with $\mathcal{LR}$ refinement

FlipFlop calculus (FFC) overview

short name	ffc, ff, flipflop
calculus parameters	none
arity	ternary
entity type	2D points
description	relates the referent C relative to the line segment starting at origin A and ending at relatum B resulting in seven base relations
base relations	l (left), r (right), f (front), b (back), s (start), e (end), dou, tri
references	Ligozat (1993), Scivos and Nebel (2005)
remark	SparQ uses the $\mathcal{LR}$ refinement in its implementation of the FFC

The FlipFlop calculus proposed in Ligozat (1993) describes the position of a point C (the referent) in the plane with respect to two other points A (the origin) and B (the relatum) as illustrated in Fig. 4.1. It can for instance be used to describe the spatial relation of C to B as seen from A. For configurations with $A \neq B$ the following base relations are distinguished: C can be to the **l**eft or to the **r**ight of the oriented line going through A and B, or C can be placed on the line resulting in one of the five relations **i**nside, **f**ront, **b**ack, **s**tart ($C = A$) or **e**nd ($C = B$) (cp. Fig. 4.1). Relations for the case where A and B coincide were not included in Ligozat's original definition (Ligozat, 1993). This was done with the $\mathcal{LR}$ refinement (Scivos and Nebel, 2005) that introduces the relations **dou** ($A = B \neq C$) and **tri** ($A = B = C$) as additional relations, resulting in 9 base relations overall. A $\mathcal{LR}$ relation $rel_{\mathcal{LR}}$ is written as $A, B\ rel_{\mathcal{LR}}\ C$, e.g. $A, B\ \mathbf{r}\ C$ as depicted in Fig. 4.1.

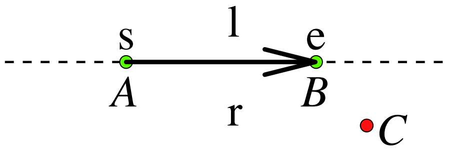


Figure 4.1: The reference frame for the $\mathcal{LR}$ calculus, an enhanced version of the FlipFlop Calculus

4.2 Singlecross calculus (SCC)

Singlecross calculus (SCC) overview

short name	scc
calculus parameters	none
arity	ternary
entity type	2D points
description	relates the referent c relative to the line between origin a and relatum b and the orthogonal line thru b, resulting in nine relations
base relations	0..7 and b (for b=c)
references	Freksa (1992a)

The single cross calculus is a ternary calculus that describes the direction of a point C (the referent) with respect to a point B (the relatum) as seen from a third point A (the origin). It has originally been proposed in Freksa (1992a). The plane is partitioned into regions by the line going through A and B and the perpendicular at B. This results in eight possible directions for C as illustrated in Fig. 4.2. We denote these base relations by numbers from 0 to 7 instead of using linguistic prepositions, e.g. 2 instead of *left*, as originally done in Freksa (1992a). Relations 0, 2, 4, 6 are linear ones, while relations 1, 3, 5, 7 are planar. In addition, three special relations exist for the cases $A \neq B = C$ (**bc**), $A = B \neq C$ (**dou**), and $A = B = C$ (**tri**). A single cross relation rel_{SCC} is written as $A, B\ rel_{SCC}\ C$, e.g. $A, B\ \mathbf{4}\ C$ or $A, B\ \mathbf{dou}\ C$. The relation depicted in Fig. 4.2 is the relation $A, B\ \mathbf{5}\ C$.

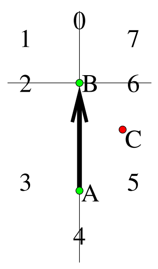


Figure 4.2: The Single Cross Reference System

4.3 Doublecross calculus (DCC)

Doublecross calculus (DCC) overview

short name	dcc, double-cross
calculus parameters	none
arity	ternary
entity type	2D points
description	relates the referent C relative to the line between origin A and relatum B and the orthogonal lines through A and B, resulting in 17 base relations
base relations	0_4, 1_5, 2_5, 3_5, 3_6, 3_7, 4_0, 5_1, 5_2, 5_3, 6_3, 7_3, 4_4, 4_b, 4_a
references	Freksa (1992a)

The double cross calculus (Freksa, 1992a) can be seen as an extension of the single cross calculus adding another perpendicular, this time at A (see Fig. 4.3 (right)). It can also be interpreted as the combination of two single cross relations, the first describing the position of C with respect to B as seen from A and the second with respect to A as seen from B (cf. Fig. 4.3 (left)). The resulting partition distinguishes 13 relations (7 linear and 6 planar) denoted by tuples derived from the two underlying SCC reference frames and four special cases, $A = C \neq B$ (**4_a**), $A \neq B = C$ (**b_4**), $A = B \neq C$ (**dou**), and $A = B = C$ (**tri**), resulting in 17 base relations overall. In Fig. 4.3 the relation A, B **5_3** C is depicted.

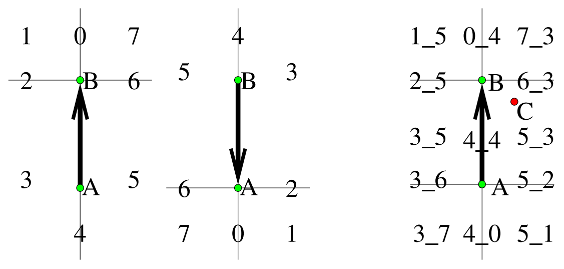


Figure 4.3: The two Single Cross reference frames resulting in the overall Double Cross Calculus reference frame

4.4 Dipole calculus family

A dipole is an oriented line segment as e.g. determined by a start and an end point. We will write $\vec{d}_{AB}$ for a dipole defined by start point A and end point B. The idea of using dipoles was first introduced by Schlieder (1995) and extended resulting in the coarse-grained Dipole Relation Algebra $\mathcal{DRA}_c$ (Moratz et al., 2000). Later, a fine-grained version of the dipole calculus ($\mathcal{DRA}_f$) has been proposed (Dylla and Moratz, 2005) and which has further been extended to $\mathcal{DRA}_{fp}$ (Dylla and Moratz, 2005). In SparQ, currently only the coarse-grained version $\mathcal{DRA}_c$ is available.

Coarse-grained Dipole Relation Algebra ($\mathcal{DRA}_c$)

Coarse-grained dipole calculus ($\mathcal{DRA}_c$) overview

short name	dra-24, dipole-coarse
calculus parameters	none
arity	binary
entity type	dipoles in the plane (oriented line segments)
description	relates two dipoles using the FlipFlop relations between the start and end point of one dipole and the other dipole
base relations	4-symbol words where each symbol can be either l (left), r (right), s (start), or e (end) (not all combinations are possible)
references	Moratz et al. (2000)

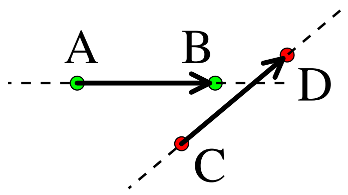


Figure 4.4: A dipole configuration: $\vec{d}_{AB}$ *rlll* $\vec{d}_{CD}$ in the coarse-grained dipole relation algebra ($\mathcal{DRA}_c$).

The coarse-grained dipole calculus variant ($\mathcal{DRA}_c$) describes the orientation relation between two dipoles $\vec{d}_{AB}$ and $\vec{d}_{CD}$ with the preliminary of A, B, C, and D being in general position, i.e. no three disjoint points are collinear. Each base relation is a 4-

tuple (r_1, r_2, r_3, r_4) of FlipFlop relations relating a point from one of the dipoles with the other dipole. r_1 describes the relation of C with respect to the dipole $\vec{d}_{AB}$, r_2 of D with respect to $\vec{d}_{AB}$, r_3 of A with respect to $\vec{d}_{CD}$, and r_4 of B with respect to $\vec{d}_{CD}$. The distinguished FlipFlop relations are **l**eft, **r**ight, **s**tart, and **e**nd (see Fig. 4.1). Dipole relations are usually written without commas and parentheses, e.g. $rrll$. Thus, the example in Fig. 4.4 shows the relation $\vec{d}_{AB}\ rlll\ \vec{d}_{CD}$. Since the underlying points for a $\mathcal{DRA}_c$ relation need to be in general position, r_i can only take the values **l**eft, **r**ight, **s**tart, or **e**nd resulting in 24 base relations.

4.5 Oriented Point Relation Algebra with granularity m ($\mathcal{OPRA}_m$)

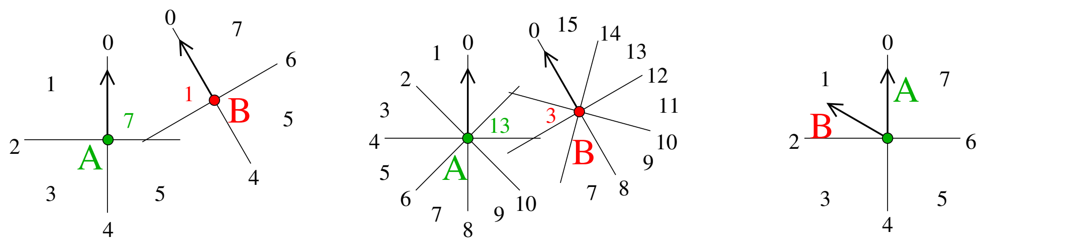
Oriented Point Relation Algebra ($\mathcal{OPRA}_m$) overview

short name	opra
calculus parameters	granularity - number of partitioning lines (= number of planar relations / 2), must be > 0
arity	binary
entity type	oriented 2D points
description	relates two oriented points a and b with respect to granularity m
base relations	$[i, j]$ with $i, j \in \{0, .., 4m - 1\}$, if a and b have different positions; $[i]$ with $i \in \{0, .., 4m - 1\}$ if they have the same position
references	Moratz et al. (2005), Moratz (2006)

The domain of the Oriented Point Relation Algebra ($\mathcal{OPRA}_m$) (Moratz et al., 2005, Moratz, 2006) is the set of oriented points (points in the plane with an additional direction parameter). The calculus relates two oriented points with respect to their relative orientation towards each other. An oriented point $\vec{O}$ can be described by its Cartesian coordinates $x_O, y_O \in \mathbb{R}$ and a direction $\phi_{\vec{O}} \in [0, 2\pi]$ with respect to an absolute reference direction and thus $D = \mathbb{R}^2 \times [0, 2\pi]$.

The $\mathcal{OPRA}_m$ calculus is suited for dealing with objects that have an intrinsic front or move in a particular direction and can be abstracted as points. The exact set of base relations distinguished in $\mathcal{OPRA}_m$ depends on the granularity parameter $m \in \mathbb{N}$. For each of the two related oriented points, m lines are used to partition the plane into $2m$ planar and $2m$ linear regions. Fig. 4.5 shows the partitions for the cases $m = 2$ (a) and $m = 4$ (b). The orientation of the two points is depicted by the arrows starting at $\vec{A}$ and $\vec{B}$, respectively. The regions are numbered from 0 to $4m - 1$; region 0 always coincides with the orientation of the point. An $\mathcal{OPRA}_m$ base relation $rel_{\mathcal{OPRA}_m}$ consists of a pair (i, j) where i is the number of the region of $\vec{A}$ which contains $\vec{B}$, while j is the number of the region of $\vec{B}$ that contains $\vec{A}$. These relations are usually written as $\vec{A}\ {}_m\angle_i^j\ \vec{B}$ with $i, j \in \mathcal{Z}_{4m}$[1]. Thus, the examples in Fig. 4.5 depict the relations $\vec{A}\ {}_2\angle_7^1\ \vec{B}$ and $\vec{A}\ {}_4\angle_{13}^3\ \vec{B}$. Additional base relations called *same relations* describe situations in which the positions of both oriented points coincide. In these cases, the relation is determined

[1] $\mathcal{Z}_{4m}$ defines a cyclic group with $4m$ elements.



(a) $m = 2$: $\vec{A}\ {}_2\angle^1_7\ \vec{B}$ (b) $m = 4$: $\vec{A}\ {}_4\angle^3_{13}\ \vec{B}$ (c) case where $\vec{A}$ and $\vec{B}$ coincide: $\vec{A}\ {}_2\angle 1\ \vec{B}$

Figure 4.5: Two oriented points related at different granularities.

by the number s of the region of $\vec{A}$ into which the orientation arrow of $\vec{B}$ falls (as illustrated in Fig. 4.5(c)). These relations are written as $\vec{A}\ {}_2\angle s\ \vec{B}$ ($\vec{A}\ {}_2\angle 1\ \vec{B}$ in the example).

The complete set $\mathcal{R}$ of $\mathcal{OPRA}_m$ relations is the power set of the base relations described above.

4.6 The Region Connection Calculus family (RCC)

The calculi from the RCC family (RCC-8 and RCC-5) allow mereotopological reasoning (reasoning about connection and part-of relationships) about simple regions in the plane. Other domains involving regions can also be considered in the context of RCC, e.g. 3D regions, or non-simple regions in the plane, which can affect the correctness of the constraint-based reasoning algorithms. Since so far no qualifier for RCC is available in SparQ, the exact domain is actually still not determined. However, we will assume the case of simple regions in the plane in the following.

RCC-8

Region Connection Calculus 8 (RCC-8) overview

short name	rcc-8
calculus parameters	none
arity	binary
entity type	simple regions in the plane
description	describes the mereotopological relation between two regions
base relations	dc (disconnected), ec (externally connected), po (partially overlapping), eq (equal), tpp (tangential proper part), ntpp (non-tangential proper part), tppi (tangential proper part inverse), nttpi (non-tangential proper part inverse)
references	Randell et al. (1992), Cohn et al. (1997)
remarks	no qualifier is available for this calculus yet

RCC-8 is the more fine-grained variant of RCC calculi. It distinguishes the eight base relations dc (disconnected), ec (externally connected), po (partially overlapping), eq (equal), tpp (tangential proper part), ntpp (non-tangential proper part), tppi (tangential proper part inverse), and nttpi (non-tangential proper part inverse) which are illustrated in Fig. 4.6.

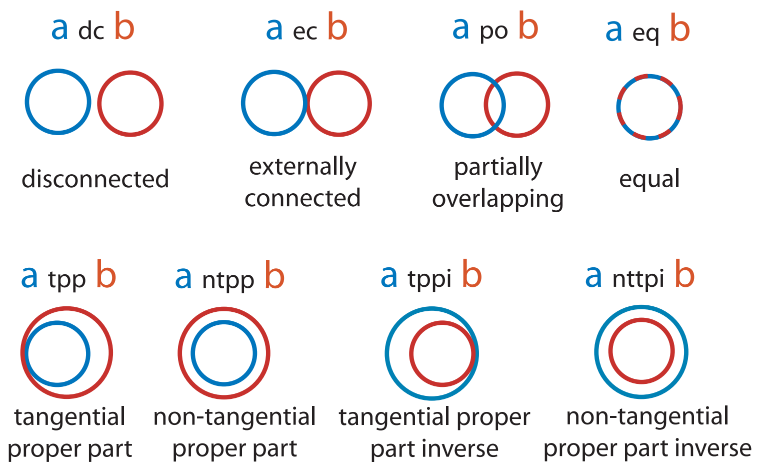


Figure 4.6: The RCC-8 base relations.

RCC-5

Region Connection Calculus 5 (RCC-5) overview

short name	rcc-5
calculus parameters	none
arity	binary
entity type	simple regions in the plane
description	describes the mereotopological relation between two regions
base relations	dr (discrete from), po (partially overlapping), eq (equal), pp (proper part), ppi (proper part inverse)
references	Cohn et al. (1997)
remarks	no qualifier is available for this calculus yet

RCC-5 is a coarser version of RCC-8. The RCC-8 relations dc and ec are combined into one relation called dr. Similarly, nttp and ttp are combined into pp and nttpi and ttpi into ppi.

4.7 Allen's Interval Algebra (IA)

Allen's Interval Algebra overview

short name	allen, aia, ia
calculus parameters	none
arity	binary
entity type	intervals (defined by a start and end-point) on a unidirectional time line
description	describes the mereotopological relation between two intervals
base relations	b (before), bi (before inverse), m (meets), mi (meets inverse), o (overlaps), oi (overlaps inverse), s (starts), si (starts inverse), d (during), di (during inverse), f (finishes), fi (finishes inverse), eq (equals)
references	Allen (1983)
remarks	no qualifier is available for this calculus yet

Allen's interval algebra (IA) (Allen, 1983) relates pairs of time intervals. A time interval x can be represented as a tuple of a starting point x_s and end point x_e ($\langle x_s, x_e \rangle$) of real numbers with $x_s < x_e$. 13 base relations are distinguished (cf. Tab. 4.2): **b**efore, **m**eets, **o**verlaps, **s**tarts, **d**uring, **f**inishes, **b**efore **i**nverse, **m**eets **i**nverse, **o**verlaps **i**nverse, **s**tarts **i**nverse, **d**uring **i**nverse, **f**inishes **i**nverse, **eq**uals. The relations can be defined by comparing the start and end points of the intervals (see Tab. 4.2). The converse operation always takes an interval relation r to the corresponding inverse relation, e.g. f to fi.

Basic relation (abbrv.)		Example	Condition
x before y	(b)	xxx	$x_e < y_s$
y after x	(bi)	yyy	
x meets y	(m)	xxxx	$x_e = y_s$
y met-by x	(mi)	yyyy	
x overlaps y	(o)	xxxxx	$x_s < y_s < x_e \wedge$
y overlapped-by x	(oi)	yyyyy	$x_e < y_e$
x during y	(d)	xxx	$x_s > y_s \wedge$
y includes x	(di)	yyyyyyy	$x_e < y_e$
x starts y	(s)	xxx	$x_s = y_s \wedge$
y started-by x	(si)	yyyyyyy	$x_e < y_e$
x finishes y	(f)	xxx	$x_e = y_e \wedge$
y finished-by x	(fi)	yyyyyyy	$x_s > y_s$
x equals y	(eq)	xxxxxxx	$x_s = y_s \wedge$
		yyyyyyy	$x_e = y_e$

Table 4.2: The 13 basic Allen relations.

5 Using SparQ

SparQ consists of a set of modules that logically structure the different services provided, which will be explained below. The general architecture is visualized in Fig. 5.1. The dashed parts are extensions planned for the future (see Section 6).

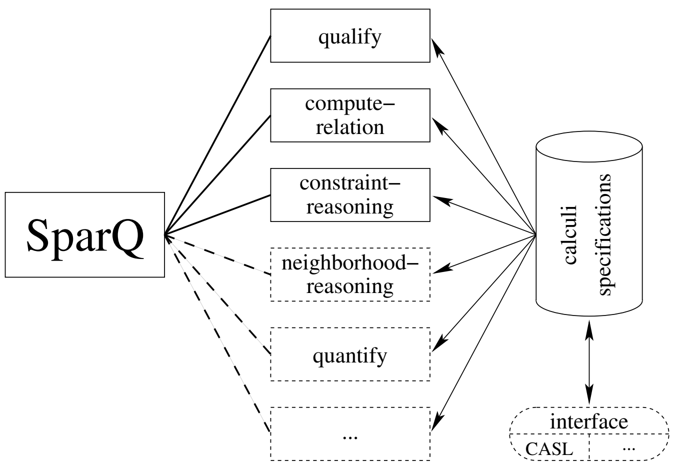


Figure 5.1: Module architecture of the SparQ toolbox.

The general syntax for using the SparQ main script which can be found in the main directory of SparQ is as follows:

```
$ ./sparq <module> <calculus identifier> <module specific parameters>
```

Example:

```
$ ./sparq compute-relation dra-24 complement "(lrll llrr)"
```

where ‘compute-relation’ is the name of the module to be utilized, in this case the module for conducting operations on relations, ‘dra-24’ is the SparQ identifier for the dipole calculus $\mathcal{DRA}_c$, and the rest are module specific parameters, here the name of the operation that should be conducted (‘complement’) and a string parameter representing the disjunction of the two dipole base relations *lrll* and *llrr*[1]. The example call thus computes the complement of the disjunction of these two relations.

[1]Unions of base relations are always represented as a space-separated list of the base relations enclosed in parentheses in SparQ.

Some calculi have calculi specific parameters, for example the granularity parameter in $\mathcal{OPRA}_m$. These parameters are appended with a '`-`' after the calculus' base identifier. `opra-3` for example refers to $\mathcal{OPRA}_3$.

SparQ provides the following modules:

qualify — transforms a quantitative geometric description of a spatial configuration into a qualitative description based on one of the supported spatial calculi

compute-relation — applies the operations defined in the calculi specifications (intersection, union, complement, converse, composition, etc.) to a set of spatial relations

constraint-reasoning — performs computations on constraint networks

We will take a closer look at each of these three modules in the next sections.

5.1 Qualify

The purpose of the qualify module is to turn a quantitative geometric scene description into a qualitative scene description composed of base relations from a particular calculus. The calculus is specified via the calculus identifier that is passed with the call to SparQ. Qualification is required for applications in which we want to perform qualitative computations over objects whose geometric parameters are known.

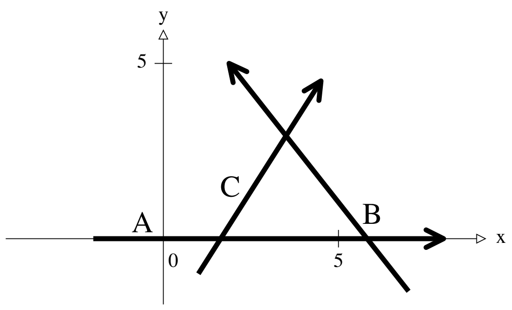


Figure 5.2: An example configuration of three dipoles.

The qualify module reads a quantitative scene description and generates a qualitative description. A quantitative scene description is a space-separated list of base object descriptions enclosed in parentheses. Each base object description is a tuple consisting of an object identifier and object parameters that depend on the type of the object. For instance, let us say we are working with dipoles which are oriented line segments. The object description of a dipole is of the form '(name x_s y_s x_e y_e)', where name is the identifier of this particular dipole object and the rest are the coordinates of start and end point of the dipole. Let us consider the example in Fig. 5.2 which shows three dipoles A, B, and C. The quantitative scene description for this situation would be:

```
( (A -2 0 8 0) (B 7 -2 2 5) (C 1 -1 4.5 4.5) )
```

The qualify module has one module specific parameter that needs to be specified:

mode — This parameter controls which relations are included into the qualitative scene description: If the parameter is ‘all’, the relation between every object and every other object will be included. If it is ‘first2all’ only the relations between the first and all other objects are computed in the binary case or between the first two objects and all other objects in the ternary case.

The resulting qualitative scene description is a space-separated list of relation tuples enclosed in parentheses. A relation tuple consists of an object identifier followed by a relation name and another object identifier, meaning that the first object stands in this particular relation with the second object. The command to produce the qualitative scene description followed by the result is[2]:

```
$ ./sparq qualify dra-24 all
$ ( (A -2 0 8 0) (B 7 -2 2 5) (C 1 -1 4.5 4.5) )
> ( (A rllr B) (A rllr C) (B lrrl C) )
```

If we had chosen ‘first2all’ as mode parameter the relation between B and C would not have been included in the qualitative scene description.

5.2 Compute-relation

The compute-relation module allows to compute with the operations defined in the calculus specification. The module specific parameters are the operation that should be conducted and one or more input relations depending on the arity of the operation. Let us say we want to compute the converse of the *llrl* dipole relation. The corresponding call to SparQ and the result are:

```
$ ./sparq compute-relation dra-24 converse llrl
> (rlll)
```

The result is always a list of relations as operations often yield a disjunction of base relations. In this case, however, the disjunction only contains a single relation. The composition of two relations requires one more relation as parameter because it is a binary operation, e.g.:

```
$ ./sparq compute-relation dra-24 composition llrr rllr
> (lrrr llrr rlrr slsr lllr rllr rlll ells llll lrll)
```

[2]In all the examples, input lines start with ‘$’. Output of SparQ is marked with ‘>’.

Here the result is a disjunction of 10 base relations. It is also possible to have disjunctions of base relations as input parameters. For instance, the following call computes the intersection of two disjunctions:

```
$ ./sparq compute-relation dra-24 intersection "(rrrr rrll rllr)"
  "(llll rrll)"
> (rrll)
```

5.3 Constraint-reasoning

The constraint-reasoning module reads a description of a constraint network—which is a qualitative scene description that may include disjunctions and may be inconsistent and/or underspecified—and performs an operation on it, e.g., a particular kind of consistency check. Which type of operation is executed depends on the first module specific parameter. Actually, four operations are implemented: 'path-consistency', 'scenario-consistency', 'refine'[3] and 'extend'.

action — The actions currently provided are 'path-consistency' and 'scenario-consistency' which determine which kind of consistency check is performed, as well as 'refine' and 'extend' which perform set operations on constraint networks.

The action 'path-consistency' causes the module to enforce path-consistency on the constraint network using van Beek's algorithm (van Beek, 1992) or detect the inconsistency of the network in the process. In case of ternary calculus the canonical extension of van Beek's algorithm as described in Dylla and Moratz (2004) is used. We could for instance check if the scene description generated by the qualify module in Section 5.1 is path-consistent—which of course it is. To make it slightly more interesting, we add the base relation *ells* to the constraint between A and C resulting in a constraint network that is not path-consistent:

```
$ ./sparq constraint-reasoning dra-24 path-consistency
$ ( (A rllr B) (A (ells rllr) C) (B lrrl C) )
> Modified network.
> ( (B (lrrl) C) (A (rllr) C) (A (rllr) B) )
```

The result is a path-consistent constraint network in which *ells* has been removed. The output 'Modified network' indicates that the original network was not path-consistent and had to be changed. Otherwise, the result would have started with 'Unmodified network'. In the next example we remove the relation *rllr* from the disjunction between A and C. This results in a constraint network that cannot be made path-consistent which implies that it is not globally consistent.

[3]In former versions of SparQ, the 'refine' action was called 'merge'. This naming is still valid for backward compatibility reasons but may vanish in the future.

```
$ ./sparq constraint-reasoning dra-24 path-consistency
$ ( (A rllr B) (A ells C) (B lrrl C) )
> Not consistent.
> ( (B (lrrl) C) (A () C) (A (rllr) B) )
```

SparQ correctly determines that the network is inconsistent and returns the constraint network in the state in which the inconsistency showed up (indicated by the empty relation () between A and C).

In a last path-consistency example we use the ternary double cross calculus:

```
$ ./sparq constraint-reasoning dcc path-consistency
$ ( (A B (7_3 6_3) C) (B C (7_3 6_3 5_3) D) (A B (3_6 3_7) D) )
> Not consistent.
> ( (A B (3_6 3_7) D) (A B (6_3 7_3) C) (B C (5_3 6_3 7_3) D) (D C () A) )
```

If 'scenario-consistency' is provided as argument, the constraint-reasoning module checks if a path-consistent scenario exists for the given network. It uses a backtracking algorithm to generate all possible scenarios and checks them for path-consistency as described above. A second module specific parameter determines what is returned as the result of the search:

return — This parameter determines what is returned in case of a constraint network for which path-consistent scenarios can be found. It can take the values 'first' which returns the first path-consistent scenario, 'all' which returns all path-consistent scenarios, and 'interactive' which returns one solution and allows to ask for the next solution until all solutions have been iterated.

Path-consistency is also used as a forward-checking method during the search to make it more efficient. For certain calculi, the existence of a path-consistent scenario implies global consistency. However, this again has to be investigated for each calculus. As a future extension it is planned to allow to specify splitting subsets of a calculus for which path-consistency implies global consistency and provide a variant of the backtracking algorithm that decides global consistency by searching for path-consistent instantiations that only contain relations from the splitting subset.

In the following example, we use 'first' as additional parameter so that only the first solution found is returned:

```
$ ./sparq constraint-reasoning dra-24 scenario-consistency first
$ ( (A rele C) (A ells B) (C errs B) (D srsl C) (A rser D) (D rrrl B) )
> ( (B (rlrr) D) (C (slsr) D) (C (errs) B) (A (rser) D) (A (ells) B)
  (A (rele) C) )
```

In case of an inconsistent constraint network, SparQ returns ‘Not consistent.’ as in the following example:

```
$ ./sparq constraint-reasoning dra-24 scenario-consistency first
$ ( (A rele C) (A ells B) (C errs B) (D srsl C) (A rser D) (D rllr B) )
> Not consistent.
```

The action ’refine’ returns the disjunction of two constraint networks. Analogously, ’extend’ returns the conjunction:

```
$ ./sparq constraint-reasoning dra-24 refine
$ ( (A (rele errs) B) ) ( (A errs B) )
> ( (A errs B) )
```

```
$ ./sparq constraint-reasoning dra-24 extend
$ ( (A rele B) ) ( (A errs B) )
> ( (A (rele errs) B) )
```

5.4 Including SparQ into own applications

SparQ can also run in server mode which makes it easy to integrate it into own applications. We have chosen a client/server approach as it allows for straightforward integration independent of the programming language used for implementing the application.

When run in server mode, SparQ takes TCP/IP connections and interacts with the client via simple plain-text line-based communication. This means the client sends commands which consist of everything following the ‘./sparq’ in the examples in this text, and can then read the results from the TCP/IP stream.

SparQ is started in server mode by providing the command line option `--interactive` (`-i`), optionally followed by `--port` (`-p`) to specify the port.

```
$ ./sparq --interactive --port 4443
```

If no port is given, SparQ interacts with standard-input and standard-output, i.e., it can be used interactively from the shell.

An example of client/server communication with SparQ is given in Listing 5.1 which shows a small Python script that opens a connection to the server and performs some simple computations (qualification, adding another relation, checking for path-consistency). It produces the following output:

```
> ( (A rrll B) (A rrll C) )
> ( (A rrll B) (A rrll C) (B eses C) )
> Not consistent.
> ( (B (eses) C) (A () C) (A (rrll) B) )
```

```
# connect to sparq server on localhost, port 4443
sock = socket.socket(socket.AF_INET, socket.SOCK_STREAM)
sock.connect(('localhost', 4443))
sockfile = sock.makefile('r')
# qualify a geometrical scenario with DRA-24
sock.send('qualify dra-24 first2all')
sock.send('((A 4 6 9 0.5) (B -5 5 0 2) (C -4 5 6 0))')
scene = readline()   # read the answer
print scene
# add an additional relation (B eses C)
sock.send('constraint-reasoning dra-24 refine')
sock.send(scene + '(B eses C)')
scene2 = readline()   # read the answer
print scene2
# check the new scenario for consistency
sock.send('constraint-reasoning dra-24 path-consistency')
sock.send(scene2)
print readline()   # print the answer
print readline()   # print the resulting constraint network
```

Listing 5.1: Integrating SparQ into own applications: an example in Python

5.5 Specifying calculi in SparQ

For most calculi it should be rather easy to include them into SparQ. The main thing that has to be done is to provide the calculus specification. Listing 5.2 shows an extract of the definition of a simple exemplary calculus for reasoning about distances between three point objects distinguishing the three relations 'closer', 'farther', and 'same'. The specification is done in Lisp-like syntax.

The arity of the calculus, the base relations, the identity relation and the different operations have to be specified, using lists enclosed in parentheses (e.g. when an operation returns a disjunction of base relations). In this example, the inverse operation applied to 'same' yields 'same' and composing 'closer' and 'same' results in the universal relation written as the disjunction of all base relations. It is not required to specify the homing, inverse short cut, and inverse homing operations (cmp. Section 3.4) as these can be computed by applying the other operations (e.g. inverse of short cut yields inverse short cut). It is principally possible to leave more operations unspecified. However, this may mean that certain computations cannot be performed for this calculus.

In addition to the calculus specification, it is necessary to provide the implementation of a qualifier function which for an n-ary calculus takes n geometric objects of the corresponding base type as input and returns the relation holding between these objects. The qualifier function encapsulates the methods for computing the qualitative relations from quantitative geometric descriptions. If it is not provided, the qualify module will not work for this calculus.

```
(def-calculus "Relative distance calculus (reldistcalculus)"
    :arity :ternary
    :base-relations (same closer farther)
    :identity-relation same
    :inverse-operation ((same same)
                        (closer closer)
                        (farther farther))
    :shortcut-operation ((same same)
                         (closer farther)
                         (farther closer))
    :composition-operation ((same same (same closer farther))
                            (same closer (same closer farther))
                            (same farther (same closer farther))
                            (closer same (same closer farther))
                            (closer closer (same closer farther))
                            (closer farther (same closer farther))
                            (farther same (same closer farther))
                            (farther closer (same closer farther))
                            (farther farther (same closer farther))))
```

Listing 5.2: Specification of a simple ternary calculus for reasoning about distances.

For some calculi, it is not possible to provide operations in form of simple tables as in the example. For instance, $\mathcal{OPRA}_m$ has an additional parameter that specifies the granularity and influences the number of base relations. Thus, the operations can only be provided in procedural form, meaning the result of the operations are computed from the input relations when they are required. For these cases, SparQ allows to provide the operations as implemented functions and uses a caching mechanism to store often required results.

6 Outlook - Planned Extensions

Besides extending the set of supported qualitative calculi, the following extensions are currently planned for the future:

neighborhood-based reasoning — a module for reasoning based on the notion of conceptual neighborhood (Freksa, 1992b) that could for instance be used for constraint relaxation and spatial planning

quantification — a module that analogous to the qualify module takes a consistent qualitative scene description and turns it into a prototypical quantitative scene description

geometric reasoning based on Gröbner bases — a module intended for calculus developers that for instance allows to derive composition tables automatically from algebraic descriptions of the base relations of the calculus (cmp. Moratz et al., 2000)

interfaces & XML specification — interfaces to exchange calculus specifications with other QSR frameworks (e.g. Wölfl and Mossakowski, 2005) and a general XML format for the specification of qualitative calculi

Further goals are to continue the optimization of the algorithms employed in SparQ, for instance by applying maximal tractable subsets for the constraint reasoning part, and to include new results from the QSR community as they become available, in particular with respect to constraint reasoning techniques for calculi for which the standard algebraic closure algorithms are insufficient to decide consistency.

Again, please feel free to contact us if you have any ideas or wishes concerning the extension or improvement of SparQ.

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